

Inverse Trig Functions

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Warmup

Find the exact value

$$1) \sec \left(\sin^{-1} \left(-\frac{1}{4} \right) \right) \frac{4\sqrt{15}}{15}$$

$$2) \sec \left(\sin^{-1} \left(-\frac{1}{2} \right) \right) \frac{2\sqrt{3}}{3}$$

$$3) \tan \left(\cos^{-1} \left(-\frac{1}{3} \right) \right) -2\sqrt{2}$$

$$4) \tan \left(\cos^{-1} \left(-\frac{1}{4} \right) \right) -\sqrt{15}$$

$$5) \csc \left(\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right) 2$$

$$6) \sec \left(\sin^{-1} \left(-\frac{\sqrt{2}}{2} \right) \right) \sqrt{2}$$

6.1 - Inverse Trigonometric Functions

Inverse

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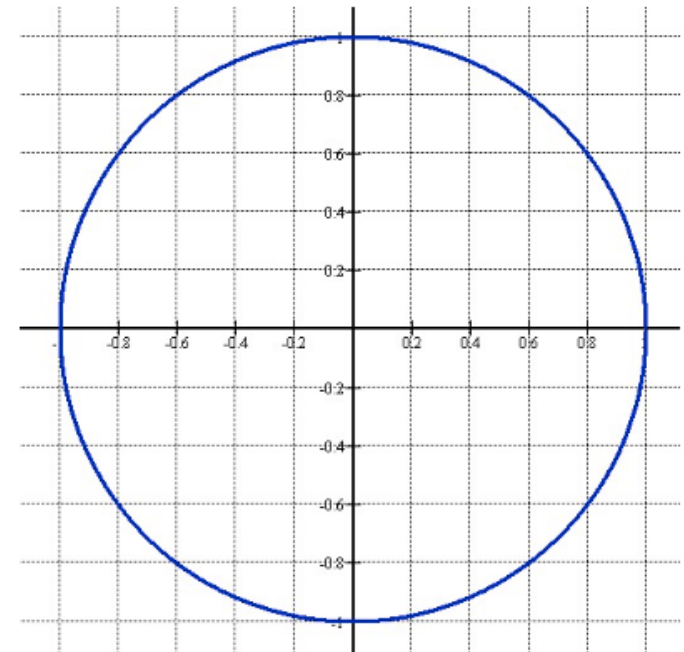
$$F(\theta) = \text{Tan}\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$F(\theta) = \text{Sin}\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$F(\theta) = \text{Cot}\theta \quad 0 < \theta < \pi$$

$$F(\theta) = \text{Cos}\theta \quad 0 \leq \theta \leq \pi$$

How to remember these constraints



6.1 - Inverse Trigonometric Functions

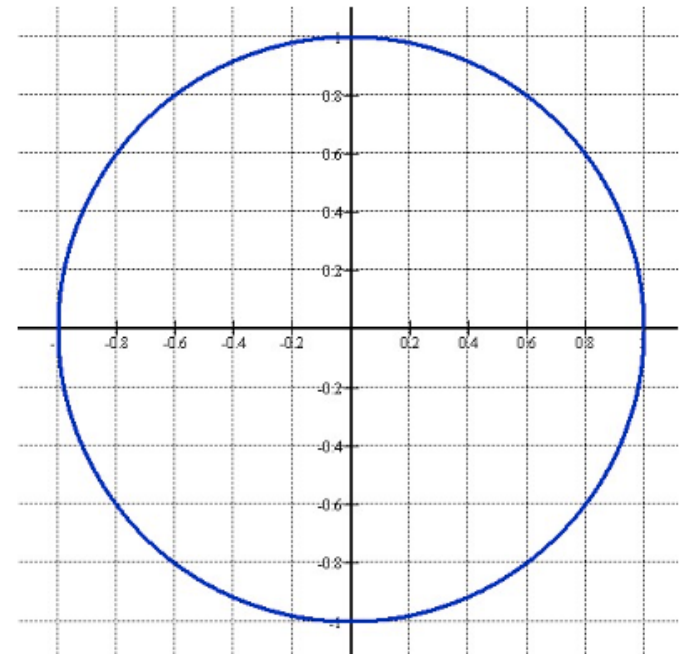
Inverse

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$$x = \cos^{-1}(y) + \sin^{-1}(y)$$

$$x = \theta + (90^\circ - \theta)$$

$$x = 90^\circ$$



6.1 - Inverse Trigonometric Functions

Inverse

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$$x = \cos \left(\tan^{-1} \left(\frac{12}{5} \right) + \sin^{-1} \left(\frac{3}{5} \right) \right)$$

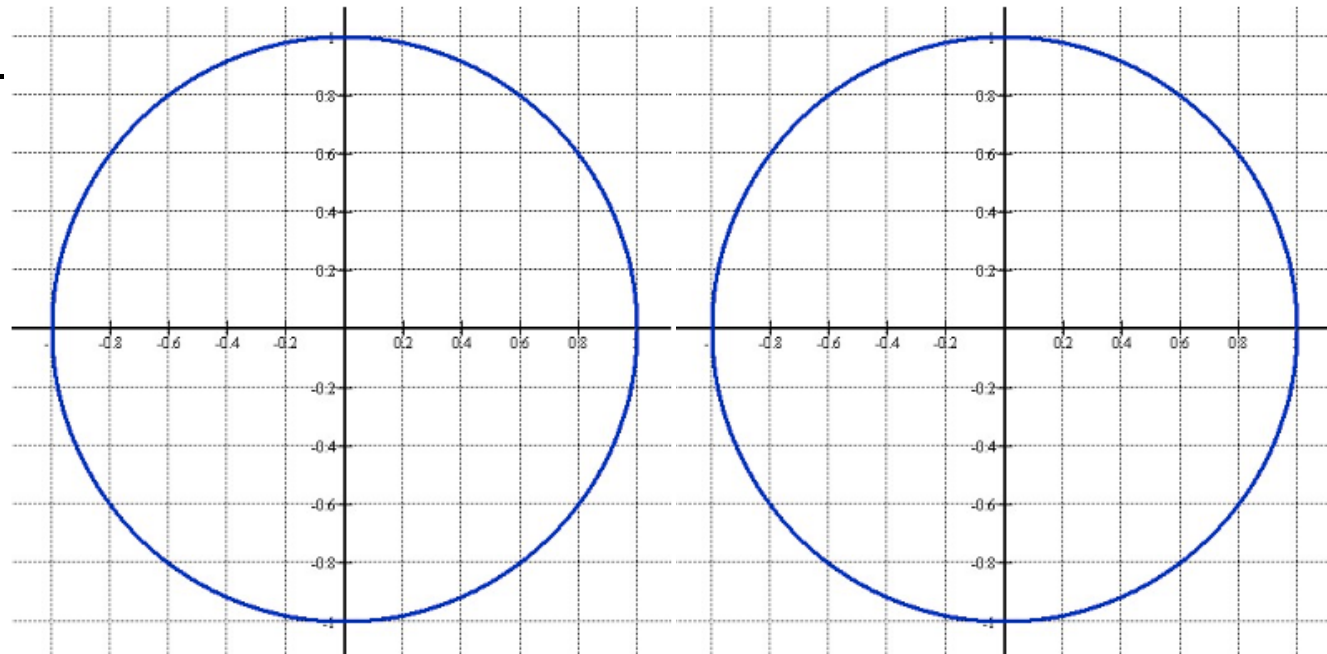
$$x = \cos(A + B)$$

$$x = \cos A \cos B - \sin A \sin B$$

$$x = \frac{5}{13} \cdot \frac{4}{5} - \frac{12}{13} \cdot \frac{3}{5}$$

$$x = \frac{20}{65} - \frac{36}{65}$$

$$x = -\frac{16}{65}$$



6.1 - Inverse Trigonometric Functions

Inverse

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Practice - Evaluate in radians

$$3 \cos^{-1} (2 - x^2) + 3 \sin^{-1} (2 - x^2)$$

$$\frac{3\pi}{2}$$

6.1 - Inverse Trigonometric Functions

Inverse

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Practice - Simplify

$$3 \left(\cos \left(-2 \sin^{-1}(\cos x) \right) \right) \\ 3 \left(\cos \left(-2 \sin^{-1}(\sin(90^\circ - x)) \right) \right)$$

$$3 \sin^2 x - 3 \cos^2 x$$

6.1 - Inverse Trigonometric Functions

Inverse

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Practice - Solve for x where $0^\circ \leq x < 360^\circ$

1) $\sin^2 x = 0.85$

$x = 67.2^\circ, 112.8^\circ,$
 $247.2^\circ, 292.8^\circ$

2) $\cot^{-1} \left(\cos \left(\sin^{-1} x \right) \right) = 64.74^\circ$

$x = \pm 0.88$

Chapter 6

Solving Trigonometric Equations

1. Inverse Trigonometric Functions
2. **Solving One Trig Function**
3. Solving Multiple Trig Functions

6.2 - Solving One Trig Function

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Solving equations

$$\cos(2A) = \cos^2 A - \sin^2 A \quad \cos(2A) = 1 - 2 \sin^2 A \quad \tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin(2A) = 2 \sin A \cos A \quad \cos(2A) = 2 \cos^2 A - 1 \quad \tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

Practice - Solve in degrees ($0^\circ < x < 360^\circ$)

1) $3 \cos(2x) + \cos x = 2$

$$3(2 \cos^2 x - 1) + \cos x = 2$$

$$x = 33.6^\circ, 180^\circ, 326.4^\circ$$

2) $\tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right) = 5$

$$\frac{1 - \cos x}{\sin x} - \frac{1 + \cos x}{\sin x} = 5$$

$$x = 158.2^\circ, 338.2^\circ$$

6.2 - Solving One Trig Function

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Solving equations

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1$$

$$\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

Practice - Solve in degrees ($0^\circ < x < 360^\circ$)

1) $\sin(2x) - \tan x = 0$

2) $\cos^2 x + \cos x = \cos(2x)$

$x = 45^\circ, 135^\circ, 225^\circ,$
 $315^\circ, 0^\circ, 180^\circ$

$x = 128.2^\circ, 231.8^\circ$

6.2 - Solving One Trig Function

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Solving equations

More Practice - Solve in radians ($0 \leq x < 2\pi$)

1) $2 \sin^2 x = 3(1 - \cos(-x))$

$$0, \frac{\pi}{3}, \frac{5\pi}{3}$$

2) $\tan x = 2 \sin x$

$$0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

3) $\sec^2 x - 2 \tan x = 4$

$$\frac{3\pi}{4}, \frac{7\pi}{4}, 1.249, 4.391$$

4) $\csc x + \cot x = 1$

$$\frac{1 + \cos x}{\sin x} = \cot \left(\frac{x}{2} \right) = 1$$

$$\frac{x}{2} = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{\pi}{2}, \frac{5\pi}{2}$$



Ch 6 Review

6.1 - Inverse Trigonometric Functions

Inverse

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Practice - Evaluate as a fraction

$$\sec \left(\cos^{-1} \left(\frac{5}{13} \right) + \sin^{-1} \left(-\frac{3}{5} \right) \right) = \frac{65}{56}$$

6.1 - Inverse Trigonometric Functions

Inverse

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Practice - Simplify

$$3 \sin(\cos^{-1}(-2 \sin(\tan^{-1} x)))$$

$$\frac{3\sqrt{(1-3x^2)(x^2+1)}}{x^2+1}$$

6.2 - Solving One Trig Function

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Practice - Solve for x where $0^\circ \leq x < 360^\circ$

1) $\cos(2x) - \sin(2x) - 2 \sin^2 x = 0$

2) $\cot^{-1}(\sin(\sec^{-1} x)) = 78^\circ$

$x = 18.43^\circ, 198.43^\circ,$
 $135^\circ, 315^\circ$

$x = 1.02$

